Chapter 7: Trig Equations and Identities Test

Multiple Choice
Identify the choice that best completes the statement or answers the question.

____ **1.** What is the exact value of the expression $\sin 280° \cos 130° - \cos 280° \sin 130°$?

A. $-\frac{1}{\sqrt{3}}$
B. $\frac{1}{\sqrt{3}}$
C. $-\frac{1}{2}$
D. $\frac{1}{2}$

____ **2.** What are the exact roots of the equation $\tan x = \sqrt{3}$ for $0 \le x \le 2\pi$?

A. $x = \frac{-2\pi}{3}$ or $x = \frac{5\pi}{6}$
B. $x = \frac{-2\pi}{3}$ or $x = \frac{\pi}{3}$
C. $x = \frac{\pi}{3}$ or $x = \frac{4\pi}{3}$
D. $x = \frac{5\pi}{6}$ or $x = \frac{7\pi}{6}$

____ **3.** Assume x is an angle in standard position with $\cos x = -\frac{1}{2}$.
In which quadrant could the terminal arm of angle x lie?

A. Quadrant 3 or 4
B. Quadrant 1 or 4
C. Quadrant 2 or 3
D. Quadrant 2 or 4

____ **4.** What are the solutions of the equation $\tan x = -\frac{1}{2}$ for $0 \le x \le 2\pi$, to the nearest hundredth?

A. $x \doteq -0.55$
B. $x \doteq 1.11$ or $x \doteq 2.68$
C. $x \doteq 2.68$ or $x \doteq 5.82$
D. $x \doteq 153.43$

____ **5.** What is the exact value of the expression $\frac{2\tan\left(\frac{\pi}{8}\right)}{1-\tan^2\left(\frac{\pi}{8}\right)}$?

A. $\frac{1}{\sqrt{2}}$
B. -1
C. $-\frac{1}{\sqrt{2}}$
D. 1

____ **6.** Write the expression $\tan^2\theta - \sec^2\theta - 3$ as a single term.

A. –4
B. –1
C. –3
D. –2

____ **7.** What are the non-permissible values of θ for the expression $\sin\theta\left(-\cot^2\theta - 1\right)$?

A. $\theta \neq \frac{\pi}{2} + \pi k, k \in \mathbf{Z}$
B. $\theta \neq \pi k, k \in \mathbf{Z}$
C. All real values are permissible.
D. $\theta \neq \frac{\pi k}{2}, k \in \mathbf{Z}$

____ **8.** Which of these values of x is NOT a solution of the equation $\cos x = \frac{1}{2}$?

A. $x = \frac{\pi}{3}$
B. $x = \frac{5\pi}{3}$
C. $x = \frac{-\pi}{3}$
D. $x = \frac{7\pi}{6}$

____ **9.** Write the expression $\sin 2\theta\cos\theta + \cos 2\theta\sin\theta$ as a single term.

A. $\cos\theta$
B. $\sin\theta$
C. $\sin 3\theta$
D. $\cos 3\theta$

____ **10.** The first two positive roots of the equation $\sin 3x = -\frac{2}{3}$ are approximately 1.29 and 1.85. Which expression represents the general solution, where $k \in \mathbf{Z}$?

A. $x \doteq 1.29 + \frac{1}{3}k$ or $x \doteq 1.85 + \frac{1}{3}k$
B. $x \doteq 1.29 + \frac{\pi}{3}k$ or $x \doteq 1.85 + \frac{\pi}{3}k$
C. $x \doteq 1.29 + \frac{2}{3}k$ or $x \doteq 1.85 + \frac{2}{3}k$
D. $x \doteq 1.29 + \frac{2\pi}{3}k$ or $x \doteq 1.85 + \frac{2\pi}{3}k$

____ **11.** Write the expression $\frac{\sin^2\theta}{\tan^2\theta}$ as a single term.

A. $\tan^2\theta$
B. $\sin^2\theta$
C. $\cos^2\theta$
D. $\sec^2\theta$

____ **12.** Use a graph to determine which of these values of x is an approximate solution of the equation $5\tan x = 3$.

A. $x \doteq 3.68$
B. $x \doteq 1.33$
C. $x \doteq 0.68$
D. $x \doteq 30.96$

Short Answer

1. Write the expression $\frac{-\sin\theta + \csc\theta}{\cot\theta}$ as a single term.

2. Solve the equation $\cos x - 5 = 6x$ over the set of real numbers. Give the answer to the nearest hundredth.

3. The first two positive roots of the equation $\sin 3x = \frac{1}{8}$ are $x \doteq 0.04$ and $x \doteq 1.01$.
Determine the general solution of this equation.

4. Determine the exact value of $\sin\frac{\pi}{12}$.

5. Given angle θ in standard position with its terminal arm in Quadrant 3 and $\cos\theta = -\frac{3}{5}$, determine the exact value of $\tan 2\theta$.

Problem

1. Use algebra to solve the equation $-\sin x = 2\cos 2x$ over the domain $0 \le x \le 2\pi$. Give the answers to the nearest hundredth.

2. For the identity $\frac{\sin\theta\cot\theta}{\cos\theta} = 1$:
a) Verify the identity for $\theta = \frac{\pi}{6}$.
b) Prove the identity.

3. Prove the identity $\frac{\csc\theta - \cot\theta}{1 - \cos\theta} = \csc\theta$.

4. Prove the identity $\frac{\cot^2\theta - \cos^2\theta}{\cos\theta\cot^2\theta + \cos^2\theta\cot\theta} = \sec\theta - \tan\theta$.

5. Prove the identity $\sin\left(\frac{\pi}{6} + \theta\right) + \sin\left(\frac{\pi}{6} - \theta\right) = \cos\theta$.

Chapter 7: Trig Equations and Identities Test
Answer Section

MULTIPLE CHOICE

1. ANS: D PTS: 1 DIF: Moderate REF: 7.5 Sum and Difference Identities
LOC: 12.T5 TOP: Trigonometry
KEY: Conceptual Understanding | Procedural Knowledge

2. ANS: C PTS: 1 DIF: Easy
REF: 7.2 Solving Trigonometric Equations Algebraically LOC: 12.T5
TOP: Trigonometry KEY: Conceptual Understanding | Procedural Knowledge

3. ANS: C PTS: 1 DIF: Easy
REF: 7.1 Solving Trigonometric Equations Graphically LOC: 12.T3
TOP: Trigonometry KEY: Conceptual Understanding

4. ANS: C PTS: 1 DIF: Moderate
REF: 7.1 Solving Trigonometric Equations Graphically LOC: 12.T5
TOP: Trigonometry KEY: Conceptual Understanding | Procedural Knowledge

5. ANS: D PTS: 1 DIF: Easy REF: 7.6 Double-Angle Identities
LOC: 12.T5 TOP: Trigonometry
KEY: Conceptual Understanding | Procedural Knowledge

6. ANS: A PTS: 1 DIF: Easy REF: 7.4 The Pythagorean Identities
LOC: 12.T6 TOP: Trigonometry KEY: Procedural Knowledge

7. ANS: B PTS: 1 DIF: Easy REF: 7.4 The Pythagorean Identities
LOC: 12.T6 TOP: Trigonometry KEY: Procedural Knowledge

8. ANS: D PTS: 1 DIF: Easy
REF: 7.2 Solving Trigonometric Equations Algebraically LOC: 12.T5
TOP: Trigonometry KEY: Procedural Knowledge

9. ANS: C PTS: 1 DIF: Easy REF: 7.5 Sum and Difference Identities
LOC: 12.T6 TOP: Trigonometry KEY: Procedural Knowledge

10. ANS: D PTS: 1 DIF: Easy
REF: 7.1 Solving Trigonometric Equations Graphically LOC: 12.T5
TOP: Trigonometry KEY: Conceptual Understanding | Procedural

Knowledge

11. ANS: C PTS: 1 DIF: Easy
REF: 7.3 Reciprocal and Quotient Identities LOC: 12.T6
TOP: Trigonometry KEY: Procedural Knowledge

12. ANS: A PTS: 1 DIF: Moderate
REF: 7.1 Solving Trigonometric Equations Graphically LOC: 12.T5
TOP: Trigonometry KEY: Procedural Knowledge

SHORT ANSWER

1. ANS:
$\cos\theta$

PTS: 1 DIF: Moderate REF: 7.4 The Pythagorean Identities
LOC: 12.T6 TOP: Trigonometry
KEY: Conceptual Understanding | Procedural Knowledge

2. ANS:
$x \doteq -0.71$

PTS: 1 DIF: Moderate REF: 7.1 Solving Trigonometric Equations Graphically
LOC: 12.T5 TOP: Trigonometry
KEY: Conceptual Understanding | Procedural Knowledge

3. ANS:
$x \doteq 0.04 + \frac{2\pi}{3}k, k \in \mathbf{Z}$ or $x \doteq 1.01 + \frac{2\pi}{3}k, k \in \mathbf{Z}$

PTS: 1 DIF: Easy REF: 7.1 Solving Trigonometric Equations Graphically
LOC: 12.T5 TOP: Trigonometry
KEY: Conceptual Understanding | Procedural Knowledge

4. ANS:
$\frac{\sqrt{3}-1}{2\sqrt{2}}$

PTS: 1 DIF: Moderate REF: 7.5 Sum and Difference Identities
LOC: 12.T5 TOP: Trigonometry
KEY: Conceptual Understanding | Procedural Knowledge

5. ANS:
$\tan 2\theta = -\frac{24}{7}$

PTS: 1 DIF: Moderate REF: 7.6 Double-Angle Identities
LOC: 12.T5 TOP: Trigonometry
KEY: Procedural Knowledge | Conceptual Understanding

PROBLEM

1. ANS:

$-\sin x = 2\cos 2x$

$-\sin x = 2\left(1 - 2\sin^2 x\right)$

$4\sin^2 x - \sin x - 2 = 0$

Use the quadratic formula: $\sin x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Substitute: $a = 4, b = -1, c = -2$

$\sin x = \dfrac{-(-1) \pm \sqrt{(-1)^2 - 4(4)(-2)}}{2(4)}$

$\sin x = \dfrac{1 \pm \sqrt{33}}{8}$

Either $\sin x = \dfrac{1 + \sqrt{33}}{8}$ or $\sin x = \dfrac{1 - \sqrt{33}}{8}$

$\dfrac{1 + \sqrt{33}}{8}$ is positive.

$\sin x$ is positive when the terminal arm of angle x in the domain $0 \le x \le 2\pi$ lies in Quadrant 1 or Quadrant 2.

The reference angle is:

$\sin^{-1}\left(\dfrac{1 + \sqrt{33}}{8}\right) = 1.0029\ldots$

In Quadrant 1, $x = 1.0029\ldots$

$\doteq 1.00$

In Quadrant 2, $x = \pi - 1.0029\ldots$

$\doteq 2.14$

$\dfrac{1 - \sqrt{33}}{8}$ is negative.

$\sin x$ is negative when the terminal arm of angle x in the domain $0 \le x \le 2\pi$ lies in Quadrant 3 or Quadrant 4.

The reference angle is:

$\sin^{-1}\left(\dfrac{-1 + \sqrt{33}}{8}\right) = 0.6348\ldots$

In Quadrant 3, $x = \pi + 0.6348\ldots$

$\doteq 3.78$

In Quadrant 4, $x = 2\pi - 0.6348\ldots$

$\doteq 5.65$

The roots are: $x \doteq 1.00$, $x \doteq 2.14$, $x \doteq 3.78$, and $x \doteq 5.65$

PTS: 1 DIF: Moderate REF: 7.6 Double-Angle Identities
LOC: 12.T5 TOP: Trigonometry
KEY: Conceptual Understanding | Procedural Knowledge | Communication

2. ANS:

a) $\dfrac{\sin\theta\cot\theta}{\cos\theta} = 1$ Substitute: $\theta = \dfrac{\pi}{6}$

$$\text{L.S.} = \frac{\sin\frac{\pi}{6}\cot\frac{\pi}{6}}{\cos\frac{\pi}{6}}$$

$$= \frac{\frac{1}{2}\cdot\sqrt{3}}{\frac{\sqrt{3}}{2}}$$

$$= 1$$

$$= \text{R.S.}$$

The left side is equal to the right side, so $\theta = \frac{\pi}{6}$ is verified.

b) $$\text{L.S.} = \frac{\sin\theta\cot\theta}{\cos\theta}$$

$$= \frac{\sin\theta\cdot\frac{\cos\theta}{\sin\theta}}{\cos\theta}$$

$$= \sin\theta\cdot\frac{\cos\theta}{\sin\theta}\cdot\frac{1}{\cos\theta}$$

$$= 1$$

$$= \text{R.S.}$$

The left side is equal to the right side, so the identity is proved.

PTS: 1 DIF: Moderate REF: 7.3 Reciprocal and Quotient Identities
LOC: 12.T6 TOP: Trigonometry
KEY: Procedural Knowledge | Conceptual Understanding | Communication

3. ANS:

R.S. $= \csc\theta$

$= \csc\theta\left(\frac{1-\cos\theta}{1-\cos\theta}\right)$

$= \frac{\csc\theta - \csc\theta\cos\theta}{1-\cos\theta}$

$= \frac{\csc\theta - \frac{1}{\sin\theta}\cdot\cos\theta}{1-\cos\theta}$

$= \frac{\csc\theta - \frac{\cos\theta}{\sin\theta}}{1-\cos\theta}$

$= \frac{\csc\theta - \cot\theta}{1-\cos\theta}$

$=$ L.S.

The left side is equal to the right side, so the identity is proved.

PTS: 1 DIF: Moderate REF: 7.3 Reciprocal and Quotient Identities
LOC: 12.T6 TOP: Trigonometry
KEY: Procedural Knowledge | Conceptual Understanding | Communication | Problem-Solving Skills

4. ANS:

L.S. $= \frac{\cot^2\theta - \cos^2\theta}{\cos\theta\cot^2\theta + \cos^2\theta\cot\theta}$

$= \frac{(\cot\theta + \cos\theta)(\cot\theta - \cos\theta)}{\cos\theta\cot\theta(\cot\theta + \cos\theta)}$

$= \frac{\cot\theta - \cos\theta}{\cos\theta\cot\theta}$

$= \frac{\cot\theta}{\cos\theta\cot\theta} - \frac{\cos\theta}{\cos\theta\cot\theta}$

$= \frac{1}{\cos\theta} - \frac{1}{\cot\theta}$

$= \sec\theta - \tan\theta$

$=$ R.S.

The left side is equal to the right side, so the identity is proved.

PTS: 1 DIF: Difficult REF: 7.3 Reciprocal and Quotient Identities
LOC: 12.T6 TOP: Trigonometry
KEY: Procedural Knowledge | Conceptual Understanding | Communication | Problem-Solving Skills

5. ANS:

$$\text{L.S.} = \sin\left(\frac{\pi}{6} + \theta\right) + \sin\left(\frac{\pi}{6} - \theta\right)$$

$$= \left(\sin\frac{\pi}{6}\cos\theta + \cos\frac{\pi}{6}\sin\theta\right) + \left(\sin\frac{\pi}{6}\cos\theta - \cos\frac{\pi}{6}\sin\theta\right)$$

$$= \sin\frac{\pi}{6}\cos\theta + \cos\frac{\pi}{6}\sin\theta + \sin\frac{\pi}{6}\cos\theta - \cos\frac{\pi}{6}\sin\theta$$

$$= 2\sin\frac{\pi}{6}\cos\theta$$

$$= 2 \cdot \frac{1}{2} \cdot \cos\theta$$

$$= \cos\theta$$

$$= \text{R.S.}$$

The left side is equal to the right side, so the identity is proved.

PTS: 1 DIF: Moderate REF: 7.5 Sum and Difference Identities
LOC: 12.T6 TOP: Trigonometry
KEY: Procedural Knowledge | Conceptual Understanding | Communication